

Zillions of Practice Problems
for
Beginning Algebra

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What This Book Is All About

In the *Life of Fred: Beginning Algebra* book, there are *Your Turn to Play* sections after each topic is presented. And each *Your Turn to Play* offers complete solutions to each question.

At the end of each chapter are six problem sets. The answers to each question are either given in the book or in the study guide, *Fred's Home Companion: Beginning Algebra*.

In addition, *Fred's Home Companion: Beginning Algebra* offers additional problems with their answers.

I thought that all those problems would be enough.

I still think all those problems are enough.

As they say in German: *Genug ist genug*.*



Some of my readers have written to me, “Enough is not enough. We want more. We want tons of problems along with completely worked out solutions. We need the drill-and-kill approach. My kids are not crying when the read Fred—how could they be learning math without crying?”

If you are a part of those *some readers*, then this *Zillions of Practice Problems* was written for you.

HOW THIS BOOK IS ORGANIZED

Life of Fred: Beginning Algebra is in twelve chapters. This book is in twelve chapters.

As you work through each chapter in *Life of Fred: Beginning Algebra*, you can do as many of the problems as you like in the corresponding chapter in this book.

* “Enough is enough.” I learned this nifty phrase when I studied German in high school.

CHAPTERS IN THIS BOOK

Each chapter in this book is divided into two parts.

- ★ The first part takes each topic and offers a zillion problems.
- ★ The second part is called the *Mixed Bag*. It consists of a variety of problems from the chapter and review problems from the beginning of the book up to that point.

COMPLETE SOLUTIONS

Every problem (gasp!) will receive a detailed solution—not just the answer.

ELIMINATING TEMPTATION

The solutions and answers are all given in the back half of the book. The first question in this book is numbered “45.” The second one is “888.” In most ordinary practice books, they are numbered, “1, 2, 3 . . .” which is really silly when you think about it. In those books, when you look up the answer to “1” you might accidentally see the answer to “2” and that would spoil all the fun.

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Chapter Four

Mixture problems

6179. Joe wants to make 30 pounds of chili seasoning that contains 30% cayenne pepper. He has one sack of mild chili seasoning that is 10% cayenne and one sack of hot chili seasoning that is 40% cayenne.

How much of each sack should he use?

6704. Darlene has some pancake mix that contains 2% sugar. She calls that her bitter mix. She has some that contains 5% sugar. She calls that her too-sweet mix.

She wants to make 60 pounds of pancake mix that has 3% sugar. How much of each mix should she use?

8975. You are a veterinarian and you need to give a marten a shot to ease its pain. You need to give it 9 cc that contains 6% morphine. You have some 4% and some 10%. How much of each should you mix together?



(Side note: It is often nice in medical work to get the mathematics right. Too little pain killer and the marten remains in pain. Too much morphine and the marten will look like:



201. We need to ship 600 cars to Freedonia, and the order specifies that 28% of those cars should have tape players.

The 40% of the older cars have tape players, and 15% of the newer cars do. How many older cars should we ship?

1850. Darlene has weak dishwashing liquid that contains 17% soap. She also has strong dishwashing liquid that contains 37% soap.

Darlene just purchased a dishwashing robot. When she read the directions, she found out that the robot would only work if you gave it 444 ounces of dishwashing liquid that contained 22% soap.

How much of each dishwashing liquid should Darlene use?

(Joe never washes his dishes because he only uses paper plates.)



Chapter Four

Second part: the *Mixed Bag*: a variety of problems from this chapter and previous material

2226. Joe likes his cereal with exactly 8% flour in it. (The rest is sugar and artificial flavorings.) He asked Darlene take his two boxes of cereal—Captain Mousebait (7% flour) and Sergeant Sugar (11% flour)—and mix them together to get 300 ounces.



How much of each cereal should Darlene use?

1928. Haydn wrote the “Sun” quartets five times as long ago as Robert Frost wrote “In the Clearing.”

Sixteen years from now, the “Sun” quartets will be four times as old as “In the Clearing.”

How old is “In the Clearing” today?

2258. Joe decided one night to read “In the Clearing.” He first read the poem at the rate of 40 words/minute. Then he decided to read it backwards. Reading backwards slowed his reading rate down to 5 words/minute.

It took him a total of 27 minutes to read the poem in both directions. How long did it take Joe to read the poem in the normal direction?

2280. Joe liked to eat jelly beans while he watched television every evening. He bought red jelly beans at a cost of \$5/lb. He bought green jelly beans at \$6/lb. He spent a total of \$488. He bought 8 more pounds of the green than of the red. How many pounds of red jelly beans did he buy?

4507. This particular cat likes some bones in his fish. (It’s a good source of calcium.) This cat prefers a fish mixture with 7% bones.

How many pounds of 5% bones should he mix with 8% bones to obtain 36 pounds of fish with 7% bones?



Chapter Five

First part: Problems on Each Topic

Solving Two Equations, Two Unknowns by Elimination

1891. Solve $\begin{cases} 7x + 3y = 43 \\ 4x - 3y = 1 \end{cases}$
2292. Solve $\begin{cases} 5x + 3y = 18 \\ -5x - 10y = 10 \end{cases}$
2904. Solve $\begin{cases} 10x + 6y = 62 \\ 8x - 2y = 2 \end{cases}$
4010. Solve $\begin{cases} 4x + 3y = 3 \\ 12x - 7y = -71 \end{cases}$
4505. Solve $\begin{cases} 6x + 6y = -18 \\ 7x + 8y = -25 \end{cases}$
4519. Solve $\begin{cases} 6x + 3y = 6 \\ 9x + 10y = 42 \end{cases}$

Union of Sets

3328. What is the union of $\{3, 4, 5\}$ and $\{4, \heartsuit, w\}$?
4280. $\{8\} \cup \{1, 2\} = ?$
4500. $\{x \mid x \text{ is an integer and } x > 4\} \cup \{y \mid y \text{ is a whole number and } y < 6\}$
600. $\{1, 2, 3\} \cup \{3, 2, 1\}$
800. $\{ \} \cup \{\odot\}$
924. For any set A, it is always true that $A \cup \{ \} = A$.
For any number x, it is always true that x times 1 equals x.
 $\{ \}$ is called the identity element for union.
1 is called the identity element for multiplication.
What is the identity element for addition?
1893. $\{y \mid y \text{ is an integer and } y < -4\} \cup \{z \mid z \text{ is a natural number}\}$

The Complete Solutions and Answers

6179. Joe wants to make 30 pounds of chili seasoning that contains 30% cayenne pepper. He has one sack of mild chili seasoning that is 10% cayenne and one sack of hot chili seasoning that is 40% cayenne.

How much of each sack should he use?

❶ Using the Let $x =$ approach:

Let $x =$ the number of pounds of mild seasoning he uses. (Since we want to find out two different things, we can let x equal either one of them.)

Then $30 - x =$ the number of pounds of hot seasoning he uses. (If he uses x pounds of the mild and 30 pounds altogether, then he has to use $30 - x$ of the hot.)

The first trick for doing mixture problems is to compute the amount of the “ingredient” in the sacks you are combining.

Then $0.1x =$ the pounds of cayenne in the mild seasoning that he uses. (10% = 0.1 He is using x pounds of which 10% is cayenne.)

Then $0.4(30 - x) =$ the pounds of cayenne contributed by the hot seasoning. (He is using $30 - x$ pounds of which 40% is cayenne.)

The second trick in mixture problems is to compute the amount of the “ingredient” in the final mixture that is desired. In this case, we want 30 pounds with 30% cayenne. So the amount of the ingredient in the final mixture is $0.3(30)$ which is 9 pounds of cayenne.

You will get a lot of practice with decimals and percents in these mixture problems. Note that to change a percent like 82.357% into a decimal, you move the decimal two places to the left and get 0.82357. The old saying: *When the percent has left, you move the decimal to the left.*

Finally, the equation is $0.1x + 0.4(30 - x) = 9$ (since the amount of ingredient (cayenne) contributed by the mild is $0.1x$ and by the hot is $0.4(30 - x)$, and the total amount in the desired mixture is 9.

distributive property	$0.1x + 12 - 0.4x = 9$
combine $0.1x$ and $-0.4x$	$-0.3x + 12 = 9$
add $0.3x$ to both sides	$12 = 9 + 0.3x$
subtract 9 from both sides	$3 = 0.3x$
divide both sides by 0.3	$10 = x$

Joe uses 10 pounds of the mild seasoning and 20 pounds (which is $30 - x$) of the hot seasoning. (The question asked how much of *each* sack.)

❷ When you fill in the Six Pretty Boxes . . .

	lbs. of cayenne	% of cayenne in the seasoning	lbs. of each seasoning used
mild seasoning	$0.1x$	10%	x
hot seasoning	$0.4(30 - x)$	40%	$30 - x$